SYDNEY GRAMMAR SCHOOL TRIAL EXAMINATION 1995

3/4 UNIT MATHEMATICS FORM VI

Time allowed: 2 hours

Exam date: 18th August, 1995

Instructions:

There will be five minutes reading time.
All questions may be attempted.
All questions are of equal value.
Part marks of questions are indicated in the boxes on the left.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.

Collection:

Each question will be collected separately.
Start each question on a new answer booklet.
If you use a second booklet for a question, place it inside the first. Don't staple.
Write your candidate number on each answer booklet.

QUESTION ONE (Start a new Answer Booklet)

Marks

$\boxed{2}$ (a) Find, to the nearest degree, the acute angle between the lines $y = \frac{1}{2}x$ and $y = 2x$.

$\boxed{3}$ (b) Find the values of x for which $\dfrac{5}{x} > 2$.

$\boxed{2}$ (c) Find:

(i) $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$,

(ii) $\displaystyle\int \frac{3}{5+x^2}\,dx$.

$\boxed{2}$ (d) Differentiate $\sin^{-1} x^2$.

$\boxed{3}$ (e) Show that the tangent to the parabola $x^2 = 4ay$ at the point $(2at, at^2)$ has equation $y = tx - at^2$.

(Exam continues next page ...)

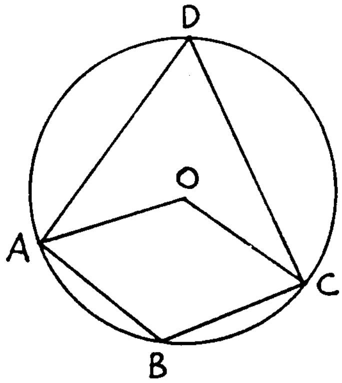
QUESTION TWO (Start a new Answer Booklet)

Marks

[3] (a) (i) Find the domain and range of the function $y = 2\cos^{-1}(x-1)$.

(ii) Sketch the graph of the function $y = 2\cos^{-1}(x-1)$.

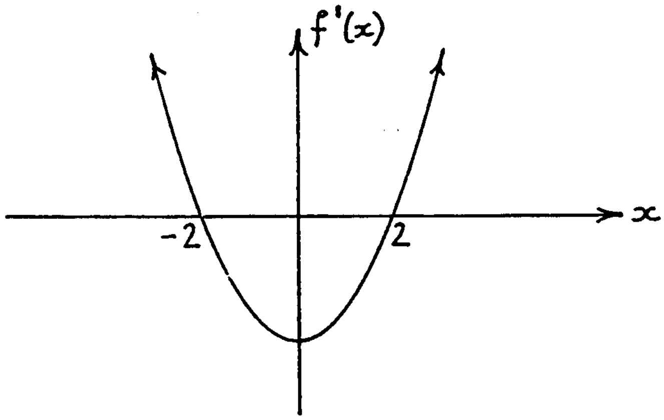
[3] (b)



O is the centre of the circle. If $\angle AOC = \angle ABC$, prove that they are both $120°$.

[3] (c) Find in exact form the volume of the solid generated when the region bounded by the curve $y = \sin 2x$ and the x axis between $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x axis.

[3] (d)



In the diagram above the gradient function $f'(x)$ of the function $y = f(x)$ is sketched. Draw a sketch of the curve $y = f(x)$ given that $f(0) = 0$.

QUESTION THREE (Start a new Answer Booklet)

Marks

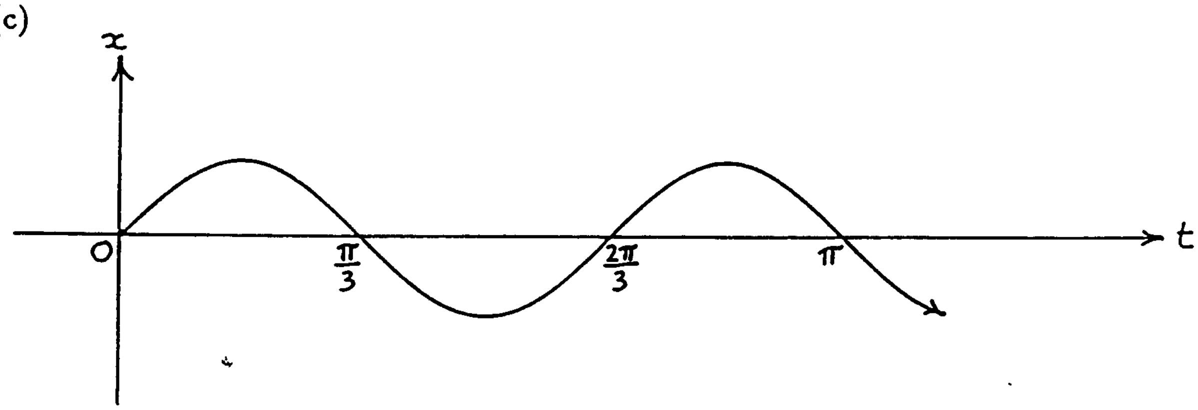
[2] (a) The equation $x^4 = 100$ has a root near $x = 3$. Use one application of Newton's method to obtain the value $3\frac{19}{108}$, which is a better approximation to this root.

[3] (b) A spherical hailstone is melting so that its volume decreases at a rate proportional to its surface area, that is $\frac{dV}{dt} = -kS$, where V is the volume at time t, S is the surface area at time t and k is a positive constant. (NOTE: $V = \frac{4}{3}\pi r^3, S = 4\pi r^2$.)

(i) Show that $\frac{dV}{dr} = S$.

(ii) Show that the radius of the hailstone is decreasing at a constant rate.

[7] (c)



The diagram shows the displacement x cm from the origin at time t seconds of a particle moving in simple harmonic motion.

(i) State the period of the motion.

(ii) At what times during the first π seconds is the particle at rest?

(iii) Show that $\ddot{x} = -9x$.

(iv) Given that the particle has initial velocity 4 cm/s, find the amplitude of the motion.

(v) Write down an equation for x in terms of t.

(Exam continues next page ...)

QUESTION FOUR (Start a new Answer Booklet)

Marks

$\boxed{4}$ (a) (i) Expand and simplify $(x+y)^6 + (x-y)^6$.

(ii) Hence, without using a calculator, show that:

$$5^6 + 5^5 \times 3^3 + 5^3 \times 3^5 + 3^6 = 2^5(2^{12}+1).$$

$\boxed{3}$ (b) By using the substitution $x = \sin\theta$, find $\int_0^{\frac{1}{2}} (1-x^2)^{-\frac{3}{2}}\,dx$.

$\boxed{5}$ (c) (i) Show that the function $T = R + Ae^{-kt}$ is a solution of the differential equation $\frac{dT}{dt} = -k(T-R)$.

(ii) A metal cake tin is removed from an oven at a temperature of 180°C. If the cake tin takes one minute to cool to 150°C and the room temperature is 20°C, find the time (to the nearest minute) it takes for the cake tin to cool to 80°C. (Assume that the cake tin cools at a rate proportional to the difference between the temperature of the cake tin and the temperature of the surrounding air.)

(iii) Sketch the graph of temperature against time for the cake tin as it cools down.

(Exam continues overleaf ...)

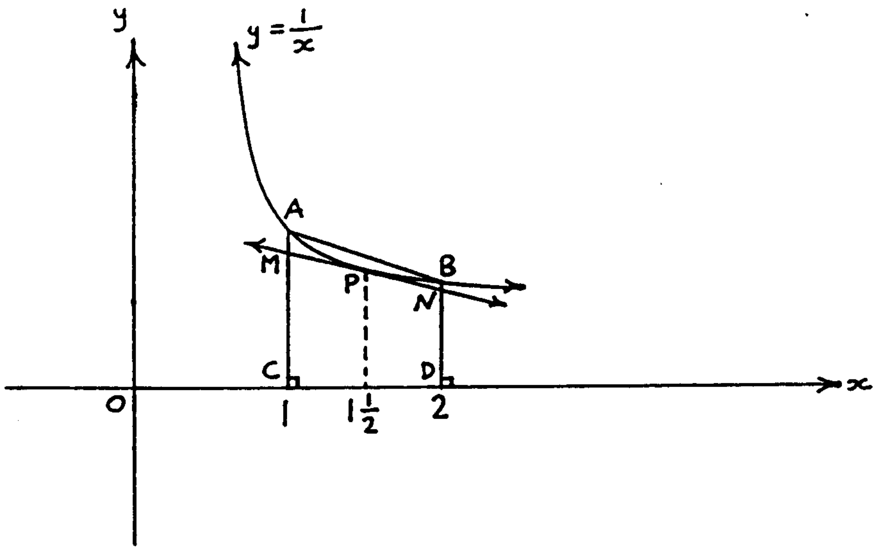
QUESTION FIVE (Start a new Answer Booklet)

Marks

$\boxed{3}$ (a) (i) Write down the term in x^r in the expansion of $(a - bx)^{12}$.

(ii) In the expansion of $(1 + x)(a - bx)^{12}$, the coefficient of x^8 is zero. Find the value of the ratio $\dfrac{a}{b}$ in simplest form.

$\boxed{6}$ (b)



The points A, P and B on the curve $y = \dfrac{1}{x}$ have x values 1, $1\frac{1}{2}$ and 2 respectively. C and D are the feet of the perpendiculars drawn from A and B to the x axis. The tangent to the curve at P cuts AC and BD at M and N respectively.

(i) Show that the tangent to the curve at P has equation $4x + 9y = 12$.

(ii) Find the coordinates of M and N.

(iii) Find the areas of the trapezia $ABDC$ and $MNDC$.

(iv) Hence show that $\frac{2}{3} < \ln 2 < \frac{3}{4}$.

$\boxed{3}$ (c) (i) Show that the arithmetic series $1 + 2 + 3 + \cdots + 24$ has a sum of 300.

(ii) Show that $\dfrac{1}{n} + \dfrac{2}{n} + \dfrac{3}{n} + \cdots + \dfrac{n}{n} = \dfrac{n+1}{2}$.

(iii) Hence find the sum of the first 300 terms of the series:

$$\frac{1}{1} + \frac{1}{2} + \frac{2}{2} + \frac{1}{3} + \frac{2}{3} + \frac{3}{3} + \frac{1}{4} + \frac{2}{4} + \frac{3}{4} + \frac{4}{4} + \cdots.$$

(Exam continues next page ...)

QUESTION SIX (Start a new Answer Booklet)

Marks

$\boxed{8}$ (a) Consider the function $y = \log_e \left(\dfrac{2x}{2+x} \right)$.

(i) Show that the domain of the function is $\{x : x > 0\} \cup \{x : x < -2\}$.

(ii) Find the value of x for which $y = 0$.

(iii) Show that $\dfrac{dy}{dx} = \dfrac{2}{x(2+x)}$ and hence show that the function is increasing for all x in the domain.

(iv) Are there any points of inflexion ?

(v) Find $\lim\limits_{x \to \infty} \left(\log_e \left(\dfrac{2x}{2+x} \right) \right)$.

(vi) Sketch the graph of the function.

$\boxed{4}$ (b) The polynomial $P(x) = x^3 - Lx^2 + Lx - M$ has roots α, $\dfrac{1}{\alpha}$ and β.

(i) Write down three equations which show the relationships between the roots and the coefficients of $P(x)$.

(ii) Show that either $M = 1$ or $M = L - 1$.

(Exam continues overleaf ...)

QUESTION SEVEN (Start a new Answer Booklet)

Marks

5 (a) (i) Prove by induction that for all positive integers n, $2^n > n$.

(ii) Hence show that $1 < \sqrt[n]{n} < 2$, if n is a positive integer greater than 1.

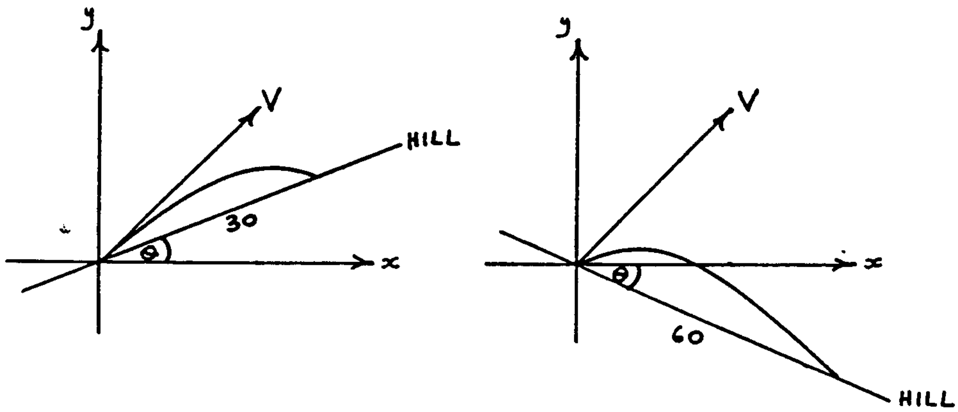
(iii) Suppose a and n are positive integers. It is known that if $\sqrt[n]{a}$ is rational then it is an integer. What can we deduce about $\sqrt[n]{n}$, where n is a positive integer greater than 1 ?

7 (b) (i) A man throws a ball with speed V m/s at an angle of 45° to the horizontal. Derive expressions for the horizontal and vertical components of the displacement of the ball from the point of projection. (Ignore air resistance.)

(ii) Hence show that the Cartesian equation of the path of the ball is given by:

$$y = x - \frac{gx^2}{V^2}.$$

(iii)



The man is now standing on a hill inclined at an angle θ to the horizontal. He throws the ball at the same angle of 45° to the horizontal and at the same speed of V m/s. If he can throw the ball 60 m down the hill but only 30 m up the hill, use the result in part (ii) to show that:

(α) $\tan\theta = 1 - \dfrac{30g\cos\theta}{V^2}$,

(β) $\tan\theta = \dfrac{60g\cos\theta}{V^2} - 1$.

(iv) Hence show that $\theta = \tan^{-1}\frac{1}{3}$.

DS

Question 1:

(a) $\tan\theta = \dfrac{2 - \frac{1}{2}}{1 + 2 \cdot \frac{1}{2}}$

$= \dfrac{3}{4}$ ✓

$\therefore \theta \doteqdot 37°$ ✓

(b) $5x > 2x^2$ ✓

$2x^2 - 5x < 0$

$x(2x-5) < 0$ ✓

$0 < x < 2\frac{1}{2}$ ✓

(c)(i) $\frac{1}{2}\log_e(1+e^{2x}) + c$ ✓

(ii) $\dfrac{3}{\sqrt{5}} \tan^{-1}\left(\dfrac{x}{\sqrt{5}}\right) + c$ ✓ (Don't penalise omission of "c")

(d) $\dfrac{2x}{\sqrt{1-x^4}}$ ✓ ✓

(e) $y = \dfrac{x^2}{4a}$

$\therefore \dfrac{dy}{dx} = \dfrac{x}{2a}$ ✓

$\therefore$ At $(2at, at^2)$, $m = \dfrac{2at}{2a} = t$ ✓

$\therefore$ Equation of tangent is

$y - at^2 = t(x - 2at)$

$y = tx - 2at^2 + at^2$

$y = tx - at^2$ ✓

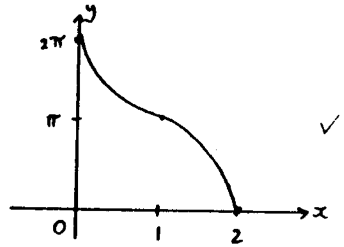
(a)(i) $-1 \le x - 1 \le 1$

$\therefore$ Domain is $\{x : 0 \le x \le 2\}$ ✓ (allow $0 \le x \le 2$)

$0 \le \frac{y}{2} \le \pi$

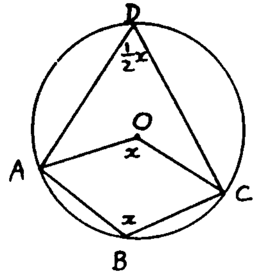
$\therefore$ Range is $\{y : 0 \le y \le 2\pi\}$ ✓

(ii)



✓

(b)



Let $\angle AOC = \angle ABC = x$

$\therefore \angle ADC = \frac{1}{2}x$ (angle at centre is twice angle at circumference) ✓

But $\angle ADC + \angle ABC = 180°$ (opposite angles of cyclic quad.) ✓

$\therefore \frac{1}{2}x + x = 180°$

$\therefore \frac{3x}{2} = 180°$

$\therefore x = 120°$ ✓

$\therefore \angle AOC = \angle ABC = 120°$ (other solutions are possible)

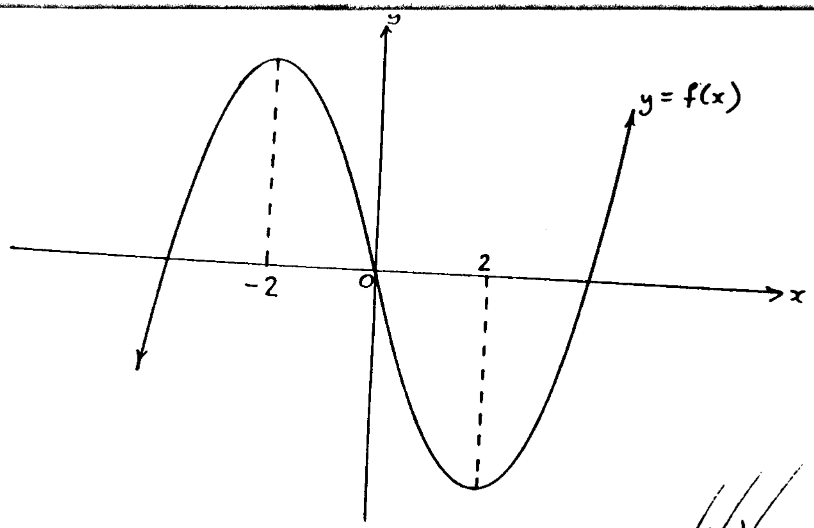
(c) $V = \pi \int_0^{\frac{\pi}{4}} \sin^2 2x \, dx$

$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} (1 - \cos 4x)\, dx$

$= \frac{\pi}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{4}}$ ✓

$= \frac{\pi}{2}\left(\frac{\pi}{4} - \frac{1}{4}\sin\pi - 0 + \frac{1}{4}\sin 0\right)$

$= \frac{\pi^2}{8}$ units^3 ✓



2 marks for correct shape,
1 mark for indicating turning points at $x = \pm 2$.

(a) Let $f(x) = x^4 - 100$
$\therefore f'(x) = 4x^3$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$
$$= 3 - \frac{f(3)}{f'(3)}$$
$$= 3 - \frac{81-100}{108}$$
$$= 3\frac{19}{108}, \text{ as required.}$$

(b)(i) $V = \frac{4}{3}\pi r^3$
$$\therefore \frac{dV}{dr} = 4\pi r^2$$
$$= S$$

(ii) By the chain rule, $\dfrac{dV}{dt} = \dfrac{dV}{dr} \cdot \dfrac{dr}{dt}$
$$\therefore -kS = S \cdot \frac{dr}{dt}$$
$$\therefore \frac{dr}{dt} = -k$$
i.e. the radius is decreasing at a constant rate.

(c)(i) Period $= \frac{2\pi}{3}$ seconds ✓ (accept $\frac{2\pi}{3}$ with no units)

(ii) Particle at rest after $\frac{\pi}{6}$, $\frac{\pi}{2}$ and $\frac{5\pi}{6}$ seconds. ✓ (don't worry about units)

(iii) Since motion is simple harmonic,

$$\left.\begin{aligned} &\ddot{x} = -n^2 x, \quad \text{where } \frac{2\pi}{n} = \frac{2\pi}{3} \\ &\therefore n = 3 \\ &\therefore \ddot{x} = -9x \end{aligned}\right\} ✓$$

(iv) $\frac{1}{2}v^2 = \int -9x \, dx$

$= \frac{-9x^2}{2} + c$ ✓

when $x = 0$, $v = 4$

$\therefore c = 8$

$\therefore v^2 = -9x^2 + 16$ ✓

At endpoints, $v = 0$

$\therefore x^2 = \frac{16}{9}$

$\therefore x = \pm\frac{4}{3}$

$\therefore$ Amplitude is $1\frac{1}{3}$ cm ✓ (accept $1\frac{1}{3}$)

(v) $x = \frac{4}{3}\sin 3t$

(a) (i) $(x+y)^6 + (x-y)^6$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$
$+ x^6 - 6x^5y + 15x^4y^2 - 20x^3y^3 + 15x^2y^4 - 6xy^5 + y^6$ ✓

$= 2x^6 + 30x^4y^2 + 30x^2y^4 + 2y^6$ ✓

(ii) Put $x = 5$, $y = 3$ into the identity in (a)

$\therefore 2(5^6 + 15.5^4.3^2 + 15.5^2.3^4 + 3^6) = 8^6 + 2^6$ ✓

$$\left.\begin{aligned} \therefore 5^6 + 5^5.3^3 + 5^3.3^5 + 3^6 &= \frac{2^{18} + 2^6}{2} \\ &= 2^{17} + 2^5 \\ &= 2^5(2^{12} + 1) \end{aligned}\right\} ✓$$

(b) $\int_0^{\frac{1}{2}} (1-x^2)^{-\frac{3}{2}} dx = \int_0^{\frac{\pi}{6}} (1-\sin^2\theta)^{-\frac{3}{2}} . \cos\theta . d\theta$

$x = \sin\theta$

$\therefore dx = \cos\theta . d\theta$

x	0	$\frac{1}{2}$
θ	0	$\frac{\pi}{6}$

✓

$= \int_0^{\frac{\pi}{6}} (\cos^2\theta)^{-\frac{3}{2}} . \cos\theta . d\theta$ ✓

$= \int_0^{\frac{\pi}{6}} (\cos\theta)^{-2} . d\theta$

$= \int_0^{\frac{\pi}{6}} \sec^2\theta . d\theta$

$= [\tan\theta]_0^{\frac{\pi}{6}}$

$= \tan\frac{\pi}{6} - \tan 0$

$= \frac{1}{\sqrt{3}}$

(d)

(a) Let $f(x) = x^4 - 100$ ✓

(c)(i) $T = R + Ae^{-kt}$

$$\therefore \frac{dT}{dt} = -k.Ae^{-kt}$$
$$= -k(T-R)$$ ✓

(ii) $R = 20$

when $t=0$, $T = 180$

$\therefore 180 = 20 + A$

$\therefore A = 160$ ✓

when $t=1$, $T=150$

$\therefore 150 = 20 + 160e^{-k}$

$\therefore e^{-k} = \frac{13}{16}$

$\therefore k = -\ln\left(\frac{13}{16}\right)$ or $\ln\left(\frac{16}{13}\right)$ ✓

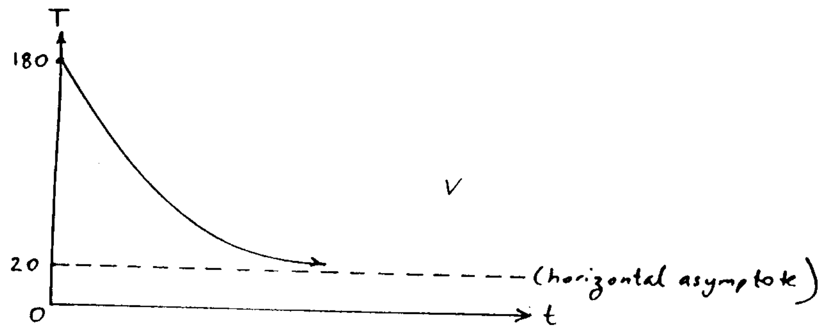
So when $T = 80$,

$80 = 20 + 160e^{-kt}$

$\therefore t = \dfrac{\ln\frac{3}{8}}{-k} = 4.7237\ldots$ ✓

So the tin takes about 5 minutes to cool to 80°C.

(iii)



✓

(a)(i) $T_{r+1} = {}^{12}C_r \cdot a^{12-r} \cdot (-bx)^r$ ✓

(ii) Coefficient of x^8 is

$${}^{12}C_8\, a^4(-b)^8 + {}^{12}C_7\, a^5(-b)^7$$ ✓

This coefficient is zero, so

$${}^{12}C_8\, a^4 b^8 = {}^{12}C_7\, a^5 b^7$$

So $\dfrac{a}{b} = \dfrac{{}^{12}C_8}{{}^{12}C_7} = \dfrac{495}{792} = \dfrac{5}{8}$ ✓

(b)(i) $y = x^{-1}$

$\therefore \dfrac{dy}{dx} = -\dfrac{1}{x^2}$

$\therefore m = \dfrac{-1}{\left(\frac{3}{2}\right)^2} = -\dfrac{4}{9}$

$\therefore$ Equation of tangent is

$y - \frac{2}{3} = -\frac{4}{9}\left(x - \frac{3}{2}\right)$

$9y - 6 = -4x + 6$

$\therefore 4x + 9y = 12$, as required. ✓

(ii) $M = \left(1, \frac{8}{9}\right)$, $N = \left(2, \frac{4}{9}\right)$ ✓ (for both)

(iii) Area of trapezium ABDC $= \frac{1}{2}\left(1 + \frac{1}{2}\right).1$

$= \frac{3}{4}$ units² ✓

Area of trapezium MNDC $= \frac{1}{2}\left(\frac{8}{9} + \frac{4}{9}\right).1$

$= \frac{2}{3}$ units² ✓

(iv) Exact area under curve from A to B $= \int_1^2 \frac{1}{x}\,dx$

$= [\ln x]_1^2$

$= \ln 2$ ✓

✓ Now, area trapezium MNDC < area under curve < area trapezium ABDC

So $\frac{2}{3} < \ln 2 < \frac{3}{4}$.

(c) (i) $S_n = \frac{n}{2}(a+l)$, where $a=1$, $l=24$, $n=24$

$\therefore S_{24} = 12(1+24)$

$= 300$

(ii) $LHS = \frac{1}{n}(1+2+3+\dots+n)$

$= \frac{1}{n} \cdot \frac{n}{2}(n+1)$

$= \frac{n+1}{2}$

$= RHS$

(iii) We want the sum of the first 24 terms of the arithmetic series $1 + 1\frac{1}{2} + 2 + 2\frac{1}{2} + \dots$

$S_{24} = 12\{2 + 23(\tfrac{1}{2})\}$

$= 162$

(a) (i) $\frac{2x}{2+x} > 0$

$2x(2+x) > 0$

$\therefore x > 0$ or $x < -2$ $\Rightarrow$ Domain is $\{x : x > 0\} \cup \{x : x < -2\}$

(ii) $y = 0$ when $\frac{2x}{2+x} = 1$

i.e. when $x = 2$

(iii) $y = \log_e 2x - \log_e(2+x)$

$\therefore \frac{dy}{dx} = \frac{1}{x} - \frac{1}{2+x}$

$= \frac{(2+x) - x}{x(2+x)}$

$= \frac{2}{x(2+x)}$, as required.

Now, $\frac{2}{x(2+x)} > 0$ for all x in the domain, since from part (i), $2x(2+x) > 0$.

$\therefore$ The function is increasing for all x in the domain.

(iv) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(x^{-1} - (2+x)^{-1}\right)$

$= -\frac{1}{x^2} + \frac{1}{(2+x)^2}$

Put $\frac{d^2y}{dx^2} = 0$ for possible points of inflexion

$\therefore \frac{1}{x^2} = \frac{1}{(2+x)^2}$

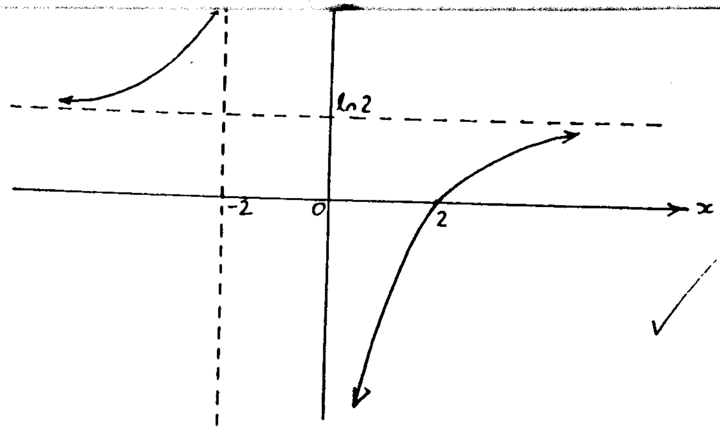
$\therefore x = -1$.

This value of x lies outside the domain, so there are no points of inflexion.

(v) $\lim_{x \to \infty}\left(\log_e\left(\frac{2}{\frac{2}{x}+1}\right)\right) = \log_e 2$

(c) (i) $T = R + Ae^{-kt}$

(a) (i) $T_{r+1} = {}^{12}C_r \cdot a^{12-r} \cdot (-bx)^r$



(b)(i) $\alpha + \frac{1}{\alpha} + \beta = L$ ①

$1 + \alpha\beta + \frac{\beta}{\alpha} = L$ ②

$\beta = M$ ③

(ii) Substitute $\beta = M$ into ①:

$\therefore\ \alpha + \frac{1}{\alpha} = L - M$ ④

Substitute $\beta = M$ into ②:

$\therefore\ \alpha M + \frac{M}{\alpha} = L - 1$

$\therefore\ \alpha + \frac{1}{\alpha} = \frac{L-1}{M}$ ⑤

From ④ and ⑤,

$\frac{L-1}{M} = L - M$

$L - 1 = ML - M^2$

$M^2 - 1 = L(M-1)$

$(M-1)(M+1) = L(M-1)$

$(M-1)(M+1-L) = 0$

from which it follows that either $M = 1$ or $M = L - 1$

(a)(i) $2^1 > 1$, so the result is true when $n = 1$.

Assume that $2^k > k$, where k is a positive integer.

Now, $2^{k+1} = 2.2^k$

$> 2k$ (using the assumption)

$= k + k$

$\geq k + 1$ (since k is a positive integer).

So the result is true for $n = k+1$ if it is true for $n = k$.

It follows that the result is true for all positive integers n, by mathematical induction.

(ii) If n is a positive integer greater than 1, it follows from part (i) that $1 < n < 2^n$.

$\therefore\ \sqrt[n]{1} < \sqrt[n]{n} < \sqrt[n]{2^n}$

$\therefore\ 1 < \sqrt[n]{n} < 2$.

(iii) $\sqrt[n]{n}$ is not an integer, since $1 < \sqrt[n]{n} < 2$.

$\therefore\ \sqrt[n]{n}$ is irrational (where n is a positive integer greater than 1).

(b)(i) $\ddot{x} = 0$

$\therefore\ \dot{x} = c_1$

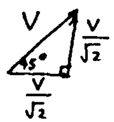
$= \frac{V}{\sqrt{2}}$

$\therefore\ x = \frac{V}{\sqrt{2}}t + c_3$

When $t = 0$, $x = 0$

$\therefore\ c_3 = 0$

$\therefore\ x = \frac{V}{\sqrt{2}}t = \frac{Vt}{\sqrt{2}}$



$\ddot{y} = -g$

$\therefore\ \dot{y} = -gt + c_2$

When $t = 0$, $\dot{y} = \frac{V}{\sqrt{2}}$

$\therefore\ c_2 = \frac{V}{\sqrt{2}}$

$\therefore\ \dot{y} = -gt + \frac{V}{\sqrt{2}}$

$\therefore\ y = -\frac{gt^2}{2} + \frac{V}{\sqrt{2}}t + c_4$

When $t = 0$, $y = 0$

$\therefore\ c_4 = 0$

$\therefore\ y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}}$

(ii) From (i), the parabolic path has parametric equation

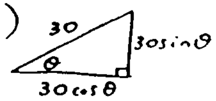
$$x = \frac{Vt}{\sqrt{2}} \quad ① \quad , \quad y = \frac{-gt^2}{2} + \frac{Vt}{\sqrt{2}} \quad ②$$

From ①, $t = \frac{\sqrt{2}\,x}{V}$

Substitute into ②: $y = \frac{-g}{2} \cdot \frac{2x^2}{V^2} + \frac{V}{\sqrt{2}} \cdot \frac{\sqrt{2}\,x}{V}$

$\therefore\ y = x - \frac{gx^2}{V^2}$, as required. ✓

(iii) (α)



The parabola passes through the point $(30\cos\theta, 30\sin\theta)$.

Substituting these coordinates into the equation of the parabola gives

$$30\sin\theta = 30\cos\theta - \frac{g \cdot 30^2 \cdot \cos^2\theta}{V^2}$$

i.e. $\tan\theta = 1 - \frac{30g\cos\theta}{V^2}$, as required. ✓

(β) In this case, the parabola passes through the point $(60\cos\theta, -60\sin\theta)$.

Substituting into the equation of the parabola gives $-60\sin\theta = 60\cos\theta - \frac{g \cdot 60^2 \cdot \cos^2\theta}{V^2}$

i.e. $\tan\theta = \frac{60g\cos\theta}{V^2} - 1$, as required. ✓

(iv) From part (iii)(α), $\frac{30g\cos\theta}{V^2} = 1 - \tan\theta$

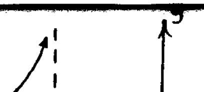
$$\frac{60g\cos\theta}{V^2} = 2 - 2\tan\theta \quad ✓$$

Substituting this into the result in part (iii)(β) gives $\tan\theta = (2 - 2\tan\theta) - 1$

$\therefore\ 3\tan\theta = 1$ ✓

$\therefore\ \theta = \tan^{-1}\frac{1}{3}$ (noting that θ is acute).

(vi)



(a)(i) $2^1 > 1$, so the result is true when $n = 1$. ✓